An alternative super-Poincare algebra

1982 J. Phys. A: Math. Gen. 15 2973

(http://iopscience.iop.org/0305-4470/15/9/042)

J. Phys. A: Math. Gen. 15 (1982) 2973–2979. Printed in Great Britain

An alternative super-Poincaré algebra

L Hlavatý
Institute of Physics, Czechoslovak Academy of Sciences, 180 40 Prague 8, Czechoslovakia

Received 3 September 1981, in final form 22 February 1982

Abstract. Properties of an alternative supersymmetric extension of the Poincaré algebra are investigated. Relations to other superalgebras are studied and the explicit form of Casimir operators is written down. Representations on a superspace as well as an analogue of the Wess–Zumino supersymmetric Lagrangian are found by a contraction procedure.

1. Introduction

In spite of the fact that the Golden Age of the classical global supersymmetry (Wess and Zumino 1974a, b, Salam and Strathdee 1974) is probably over, there are some paths which remained unnoticed. One of them is an alternative supersymmetric extension found by Konopelchenko in 1975 when he classified superalgebras containing the Lie algebra of the Poincaré group (the inhomogenous Lorentz group). Beside the well known superalgebra GL of the classical supersymmetry (Golfand and Lichtman 1971, 1972) he found another superalgebra (Konopelchenko 1975) of the same dimension but with different (anti) commutation relations of odd generators. The main reason why this algebra remained unnoticed is that it does not admit Hermitian representation of the odd operators. This leads to serious difficulties when we try to construct realistic physical models invariant under this algebra and it remained out of the classification of the S-matrix symmetries (Haag *et al* 1975). The usefulness of this superalgebra for physics is therefore questionable.

On the other hand, there are interesting relations of this superalgebra to other algebras commonly used in physics. It lies (in the sense of contractions) between the superalgebra OSp(1, 4) and the superalgebra of standard supersymmetry, and moreover, the representation of the Poincaré algebra on the odd generators of Konopelchenko's superalgebra is reminiscent of the twistorial representation of the conformal group.

Besides, most of the forthcoming results are derived by the contraction procedure and we found it interesting to see how it works in the case of non-real algebra.

Therefore, we found it reasonable, at least from the mathematical point of view, to investigate properties of Konopelchenko's superalgebra and its superfield representations.

The paper is arranged as follows. In § 2 Konopelchenko's superalgebra is defined, relations to standard supersymmetry, superconformal and Osp(1, 4) superalgebras are discussed and Casimir operators are written down. In § 3 we investigate representations on superspaces, superfields, covariant derivatives and the analogue of the Wess–Zumino supersymmetric Lagrangian.

2. Properties of Konopelchenko's superalgebra

We call the superalgebra K (Konopelchenko's) the 14-dimensional graded vector space whose basis forms ten even generators of the Poincaré algebra $J_{\mu\nu} = -J_{\nu\mu}, P_\mu$; $\mu, \nu = 0, 1, 2, 3$ with commutation relations

$$[J_{\mu\nu}, J_{\lambda\rho}] = -\mathrm{i}(\eta_{\mu\lambda}J_{\nu\rho} - \eta_{\nu\lambda}J_{\mu\rho} + \eta_{\nu\rho}J_{\mu\lambda} - \eta_{\mu\rho}J_{\nu\lambda})$$
$$[J_{\mu\nu}, P_\lambda] = -\mathrm{i}(\eta_{\mu\lambda}P_\nu - \eta_{\nu\lambda}P_\mu) \qquad [P_\mu, P_\nu] = 0 \tag{2.1}$$

where $\eta_{\mu\nu} = (+---)$ and four odd generators $Q_A, R_{\dot A}; A, \dot A = 1, 2$. The (anti) commutation relations of the odd generators are

$$[J_{\mu\nu}, Q_A] = -\tfrac{1}{2}\mathrm{i}\sigma_{\mu\nu A}{}^B Q_B \tag{2.2}$$
$$[J_{\mu\nu}, R_{\dot A}] = +\tfrac{1}{2}\mathrm{i}\overline{\sigma_{\mu\nu\dot A}{}^{\dot B}} R_{\dot B} \tag{2.3}$$
$$[P_\mu, Q_A] = 0 \tag{2.4}$$
$$[P_\mu, R_{\dot A}] = \tfrac{1}{2}a\sigma_{\mu B\dot A}Q^B \tag{2.5}$$
$$\{Q_A, Q_B\} = 0 \tag{2.6}$$
$$\{Q_A, R_{\dot B}\} = \sigma^\mu_{A\dot B}P_\mu \tag{2.7}$$
$$\{R_{\dot A}, R_{\dot B}\} = \tfrac{1}{2}a\,\mathrm{i}\overline{\sigma^{\mu\nu}}_{\dot A\dot B}J_{\mu\nu}. \tag{2.8}$$

Matrices $\sigma^\mu{}_{A\dot B}, \sigma^{\mu\nu}{}_{AB}, \overline{\sigma^{\mu\nu}}_{\dot A\dot B}$ are defined in the appendix as well as the rules for raising and lowering the indices $A, \dot A$.

In Konopelchenko's work there is one more constant b on the right-hand side of the (anti) commutations relations. It may be transformed to one when redefining $Q_A \to b^{-1/2}Q_A, R_{\dot A} \to b^{-1/2}R_{\dot A}$ (we suppose $b \neq 0$). Here, however, we keep the constant a in order to have at each step a correspondence with standard supersymmetry algebra which is obtained when $a \to 0$ (see e.g. Fayet and Ferrara 1977, Ogievetskij and Mezincescu 1975). When we proceed to the spin-tensor basis (Hlavatý and Niederle 1980) we can easily see that the involution of the Poincaré algebra

$$J^*_{\mu\nu} = J_{\mu\nu} \qquad P^*_\mu = P_\mu \tag{2.9}$$

cannot be extended to the odd elements of the superalgebra K.

Commutation relations between odd and even generators of a superalgebra generally define a representation of its even part. The representation of the Poincaré group that follows from the commutation relations (2.2)–(2.5) is identical with the Poincaré transformations of twistors (see e.g. Penrose 1975) because odd generators transform like spinors under the Lorentz transformations and the translations are represented as

$$Q_A \to Q_A \tag{2.10}$$
$$R_{\dot A} \to R_{\dot A} + \mathrm{i}ax^\mu\sigma_{\mu B\dot A}Q^B. \tag{2.11}$$

We can conclude that the *superalgebra* K *is the extension of the Poincaré algebra by odd twistorial generators.* This is related to the fact that the algebra K is a subalgebra of the superconformal algebra (see e.g. Fayet and Ferrara 1977).

It is known that the superalgebra GL is a contraction of the simple superalgebra OSp(1, 4). We shall show that the superalgebra K is also a contraction of OSp(1, 4).

The superalgebra OSp(1, 4) (Zumino 1977) has 10 even and 4 odd generators. Their (anti) commutation relations in a special choice of γ matrices and a special basis of OSp(1, 4) are

$$[J_{\mu\nu}, J_{\lambda\rho}] = -\mathrm{i}(\eta_{\mu\lambda}J_{\nu\rho} - \eta_{\nu\lambda}J_{\mu\rho} + \eta_{\nu\rho}J_{\mu\lambda} - \eta_{\mu\rho}J_{\nu\lambda}) \tag{2.12}$$

$$[J_{\mu\nu}, R_\lambda] = -\mathrm{i}(\eta_{\mu\lambda}R_\nu - \eta_{\nu\lambda}R_\mu) \tag{2.13}$$

$$[R_\mu, R_\nu] = -\mathrm{i}m^2 J_{\mu\nu} \tag{2.14}$$

$$[J_{\mu\nu}, U_A] = -\tfrac{1}{2}\mathrm{i}\sigma_{\mu\nu A}{}^{B}U_B, \qquad [J_{\mu\nu}, \bar{U}_{\dot{A}}] = -\tfrac{1}{2}\mathrm{i}\overline{\sigma_{\mu\nu\dot{A}\dot{B}}}\bar{U}^{\dot{B}} \tag{2.15}$$

$$[R_\mu, U_A] = -(c/mb)\sigma_{\mu A\dot{B}}\bar{U}^{\dot{B}}, \qquad [R_\mu, \bar{U}_{\dot{A}}] = -(b/mc)\sigma_{\mu B\dot{A}}U^{B} \tag{2.16}$$

$$\{U_A, U_B\} = \mathrm{i}b^{-2}\sigma^{\mu\nu}{}_{AB}J_{\mu\nu}, \qquad \{\bar{U}_{\dot{A}}, \bar{U}_{\dot{B}}\} = \mathrm{i}c^{-2}\overline{\sigma^{\mu\nu}}{}_{\dot{A}\dot{B}}J_{\mu\nu} \tag{2.17}$$

$$\{U_A, \bar{U}_{\dot{B}}\} = (m/bc)\sigma^{\mu}{}_{A\dot{B}}R_\mu. \tag{2.18}$$

If we want to contract the even part of OSp(1, 4) to Poincaré algebra it is necessary that $m\to\infty$. It follows from (2.18) that also $b\to\infty$ or $c\to\infty$ in order to preserve m/bc as finite. The superalgebra K is obtained by contraction $m\to\infty$, $b\to\infty$, $m/b = c = a^{-1/2}$.

The fact, that the superalgebra K is a contraction of OSp(1, 4) will be widely exploited below. The first example is the construction of Casimir operators.

Casimir operators of the superalgebra K may be obtained by contractions of OSp(1, 4) Casimir operators, the derivation of which is outlined in Jarvis and Green (1979). Detailed description of the Casimir operators construction is presented in Hlavatý and Niederle (1980), therefore in this paper we present only the results in a slightly different form. There are two independent Casimir operators of the superalgebra K which are polynomials of the second and fourth degree in the generators.

$$C_2 = P^2 - \tfrac{1}{2}aQ^2 = P_\mu P^\mu - \tfrac{1}{2}aQ^A Q_A \tag{2.19}$$

$$\begin{aligned} C_4 &= 8K^2 + \tfrac{1}{2}\{Q^2, R^2\} + a[\mathrm{i}(*J_{\mu\nu})J^{\mu\nu} + 2]Q^2 \\ &= 8K_\mu K^\mu + \tfrac{1}{2}(Q^A Q_A R_{\dot{B}}R^{\dot{B}} + R_{\dot{B}}R^{\dot{B}}Q^A Q_A) + a(\tfrac{1}{2}\mathrm{i}\varepsilon_{\mu\nu\lambda\rho}J^{\lambda\rho}J^{\mu\nu} + 2)Q^A Q_A \end{aligned} \tag{2.20}$$

$$K_\mu = \tfrac{1}{2}\varepsilon_{\mu\nu\lambda\rho}J^{\nu\lambda}P^\rho - \tfrac{1}{4}\sigma_{\mu A\dot{B}}(Q^A R^{\dot{B}} - R^{\dot{B}}Q^A). \tag{2.21}$$

It is interesting that the Casimir C_2 combines the mass operator P^2 with operator Q^2.

Therefore it might seem that multiplets of particles with different mass could exist in the framework of this alternative superalgebra. Unfortunately, as the odd elements of the superalgebra cannot be represented by Hermitian operators there are probably no particle-type representations of this superalgebra. Let us remark that the first two terms of C_4 form the fourth-order Casimir operator of the superalgebra GL that to our knowledge has never been published before.

3. Representations, superfields and K-invariant Lagrangian

The impossibility of prolonging the involution of the Poincaré algebra into the superalgebra K prevents the existence of unitary representations of K with particle interpretation. On the other hand, we can find representations of superfields i.e. functions of superspace parametrised by commuting and anticommuting coordinates.

The simplest representation may be obtained, when the Lorentz covariant ansatz

$$P_\mu = a\partial_\mu + b\theta_A\sigma_\mu^{A\dot{B}}\partial_{\dot{B}} + c\bar{\theta}_{\dot{B}}\sigma_\mu^{A\dot{B}}\partial_A \tag{3.1}$$

$$Q_A = d\partial_A + ex^\mu\sigma_{\mu A}{}^{\dot{B}}\partial_{\dot{B}} + f\bar{\theta}_{\dot{B}}\sigma^\mu{}_A{}^{\dot{B}}\partial_\mu \tag{3.2}$$

$$R_{\dot{A}} = g\partial_{\dot{A}} + hx^\mu\sigma_\mu{}^B{}_{\dot{A}}\partial_B + j\theta_B\sigma^{\mu B}{}_{\dot{A}}\partial_\mu \tag{3.3}$$

where $a, b, \ldots, j$ are functions of x^μ, is inserted into (2.4)–(2.8). It shows that in this case the superalgebra K is represented in derivations with respect to x^μ and θ_A only. No anticommuting variables $\bar{\theta}_{\dot{A}}$ are needed. Generators are represented as

$$J_{\mu\nu} = \mathrm{i}(x_\mu\partial_\nu - x_\nu\partial_\mu) - \mathrm{i}\theta_A\sigma_{\mu\nu}{}^{AB}\partial_B \tag{3.4}$$

$$P_\mu = \mathrm{i}\partial_\mu := \mathrm{i}\partial/\partial x^\mu \tag{3.5}$$

$$Q_A = \mathrm{i}\partial_A := \mathrm{i}\partial/\partial\theta^A \tag{3.6}$$

$$R_{\dot{A}} = \sigma^\mu_{B\dot{A}}(\theta^B\partial_\mu - \tfrac{1}{2}ax_\mu\partial^B). \tag{3.7}$$

Operator Q_A in this representation reminds us of 'momentum in the space of Grassman variables' while $R_{\dot{A}}$ is reminiscent of 'an angular momentum in the superspace' because it mixes commuting and anticommuting coordinates in a similar way that the usual angular momentum mixes coordinates x^μ.

Representation (3.4)–(3.7) corresponds to the realisation of the supergroup K on the coset space K/S where S is superextension of the Lorentz group by $R_{\dot{A}}$, $\mathrm{S} \propto \{J_{\mu\nu}, R_{\dot{A}}\}$. (We denote here the supergroup and its algebra by the same letter.) When we look for more complicated representations of the superalgebra K on the coset space K/O(1,3), where O(1, 3) is the Lorentz group generated by $J_{\mu\nu}$, we may again exploit the fact that the superalgebra K is the contraction of OSp(1, 4) and contract representations of OSp(1, 4) on OSp(1, 4)/O(1, 3) found by Ivanov and Sorin (1980).

The general superfield defined on K/O(1, 3) is

$$\begin{aligned}\phi_k(x^\mu, \theta^A, \eta^{\dot{A}}) = {} & A_k(x) + \theta^A\psi_{kA}(x) + \eta_{\dot{A}}\chi_k^{\dot{A}}(x) + \tfrac{1}{4}\theta^2(F_k(x) - \mathrm{i}G_k(x)) \\ & + \tfrac{1}{4}\eta^2(F_k(x) + \mathrm{i}G_k(x)) + \tfrac{1}{2}\mathrm{i}\theta^A\sigma^\mu{}_{A\dot{B}}\eta^{\dot{B}}V_{k\mu}(x) + \tfrac{1}{4}(\theta^2 + \eta^2)(\theta^A\rho_{kA}(x) \\ & + \eta_{\dot{A}}\tau_k^{\dot{A}}(x)) + \tfrac{1}{32}(\theta^2 + \eta^2)^2 D_k(x)\end{aligned} \tag{3.8}$$

where the ordinary fields $A_k(x), \ldots, D_k(x)$ carry a Lorentz group representation. Generators of the superalgebra K in the realisation of superfields are

$$J_{\mu\nu} = \mathrm{i}(x_\mu\partial_\nu - x_\nu\partial_\mu) + \tfrac{1}{2}(\theta^A\sigma_{\mu\nu A}{}^B\partial_B + \eta\dot{A}\bar{\sigma}_{\mu\nu}{}^{\dot{A}}{}_{\dot{B}}\partial^{\dot{B}}) + M_{\mu\nu} \tag{3.9}$$

$$P_\mu = \tfrac{1}{2}\mathrm{i}\partial_\mu \tag{3.10}$$

$$Q_A = \mathrm{i}(1 + \tfrac{1}{2}a\bar{\eta}^2)\partial_A - \tfrac{1}{4}\sigma^\mu{}_{A\dot{B}}\eta^{\dot{B}}\partial_\mu \tag{3.11}$$

$$\begin{aligned}R_{\dot{A}} = {} & \mathrm{i}(1 - \tfrac{1}{2}a\bar{\eta}^2)\partial_{\dot{A}} + \tfrac{1}{4}(1 - \tfrac{1}{4}a\bar{\eta}^2)\sigma^\mu{}_{\dot{A}B}\theta^B\partial_\mu \\ & + a[\tfrac{1}{2}\mathrm{i}\eta_{\dot{A}}\theta^B\partial_B - \tfrac{1}{4}M_{\mu\nu}\overline{\sigma^{\mu\nu}}{}_{\dot{A}\dot{B}}\eta^{\dot{B}} + \mathrm{i}\sigma_{\mu\dot{A}}{}^Bx^\mu Q_B]\end{aligned} \tag{3.12}$$

where $M_{\mu\nu}$ is a matrix representation of $J_{\mu\nu}$.

The representation (3.9)–(3.12) corresponds to the peculiar parametrisation of K/O(1, 3)

$$g = g(x^\mu, \theta^A, \eta^{\dot{A}}) = \exp(2\mathrm{i}x^\mu P_\mu)\exp[\mathrm{i}(1 - \tfrac{1}{3}a\eta^2)\theta^A Q_A + \mathrm{i}\eta_{\dot{A}}R^{\dot{A}}]. \tag{3.13}$$

We may, of course, choose any other parametrisation of the coset space. Parametrisations

$$g = g(x^{\mu L}, \theta^{AL}, \eta^{\dot{A}R}) = \exp(2ix_\mu^L P^\mu)\exp(i\theta^{AL}Q_A)\exp(i\eta_{\dot{A}}^R R^{\dot{A}}) \tag{3.14}$$

$$g = g(x^{\mu R}, \bar{\theta}^{\dot{A}R}, \eta^{AL}) = \exp(2ix_\mu^R P^\mu)\exp(i\bar{\theta}_{\dot{A}}^R R^{\dot{A}})\exp(i\bar{\eta}^{AL}Q_A) \tag{3.15}$$

will be useful for investigations of chiral components of superfields. Coordinate transformations induced by passage from (3.13) to (3.14) or (3.15) are

$$x_\mu^L = x_\mu - \tfrac{1}{4}i\theta^A\sigma_{\mu A\dot{B}}\eta^{\dot{B}} \qquad \theta_A^L = (1-\tfrac{1}{2}a\eta^2)\theta_A \qquad \eta_{\dot{A}}^R = \eta_{\dot{A}} \tag{3.16}$$

and

$$x_\mu^R = x_\mu + \tfrac{1}{4}i\theta^A\sigma_{\mu A\dot{B}}\eta^{\dot{B}} \qquad \bar{\theta}_{\dot{A}}^R = \eta_{\dot{A}} \qquad \bar{\eta}_A^L = \theta_A. \tag{3.17}$$

Our final goal is finding the form of a K-invariant Lagrangian similar to the Wess–Zumino (1974b) Lagrangian of the standard supersymmetry. Once more we shall contract analogous results to Ivanov and Sorin (1980) for OSp(1, 4). First of all we have to find irreducible components of the superfield (3.8). They may be obtained by virtue of covariant derivatives

$$D_A = \partial_A - \tfrac{1}{4}i\sigma_{A\dot{B}}^\mu\partial_\mu\eta^{\dot{B}} \tag{3.18}$$

$$D_{\dot{A}} = (1+\tfrac{1}{4}a\eta^2)\partial_{\dot{A}} + \tfrac{1}{4}i(1-\tfrac{1}{4}a\eta^2)\theta^B\sigma_{B\dot{A}}^\mu\partial_\mu + a(\eta^{\dot{A}}\theta^B\partial_B - \tfrac{1}{4}iM^{\mu\nu}\bar{\sigma}_{\mu\nu\dot{A}\dot{B}}\eta^{\dot{B}}). \tag{3.19}$$

The irreducible, so-called chiral, components ϕ_k^L and ϕ_k^R meet conditions

$$D_A\phi_k^R(x^\mu, \theta^A, \eta^{\dot{A}}) = 0 \tag{3.20}$$

$$D_{\dot{A}}\phi_k^L(x^\mu, \theta^A, \eta^{\dot{A}}) = 0. \tag{3.21}$$

To solve these conditions it is convenient to rewrite the covariant derivatives in coordinates (x^L, θ^L, η^R), $(x^R, \bar{\theta}^R, \bar{\eta}^L)$

$$D_A = \partial_A^L := \partial/\partial\theta_A^L \tag{3.22}$$

$$D_{\dot{A}} = (1+(a/4)\bar{\theta}^{R^2})\bar{\partial}_{\dot{A}}^R - \tfrac{1}{4}aiM_{\mu\nu}\sigma^{\mu\nu}{}_{\dot{A}\dot{B}}\bar{\theta}^{\dot{B}R}. \tag{3.23}$$

Solutions of (3.20), (3.21) are

$$\phi_k^R = \phi_k^R(x^R, \bar{\theta}^R) = A_k^R(x^R) + \bar{\theta}_{\dot{A}}^R\bar{\psi}_k^{\dot{A}R}(x^R) + \tfrac{1}{2}\bar{\theta}^{R^2}F_k^R(x^R) \tag{3.24}$$

$$\phi_k^L = \phi_k^L(x^L, \theta^L) = A_k^L(x^L) + \theta^{AL}\psi_{kA}^L(x^L) + \tfrac{1}{2}\theta^{L^2}F_k^L(x^L) \tag{3.25}$$

where, however, the left fields A_k^L, ψ_k^L, F_k^L are not arbitrary but fulfil conditions

$$\sigma^{\overline{\mu\nu}}{}_{\dot{A}\dot{B}}(M_{\mu\nu})_k{}^j X_j^L(x^L) = 0 \qquad X = A, \psi, F. \tag{3.26}$$

It means that the fields transform in index k under the representation $D(q, 0)$ of the Lorentz group (see e.g. Schweber 1961).

The form of the K-invariant Lagrangian is derived from the superfield action

$$S = S_{\text{kin}} + S_M + S_g = \int \mathcal{D}\mu\phi^R\phi^L + \tfrac{1}{2}M\left(\int \mathcal{D}\mu^L\phi^{L^2} + \int \mathcal{D}\mu^R\phi^{R^2}\right)$$
$$+\tfrac{1}{3}\sqrt{2}g\left(\int \mathcal{D}\mu^L\phi^{L^3} + \int \mathcal{D}\mu^R\phi^{R^3}\right) \tag{3.27}$$

where $\mathscr{D}\mu$, $\mathscr{D}\mu^{\mathrm{R}}$, $\mathscr{D}\mu^{\mathrm{L}}$ are K-invariant integration measures

$$\mathscr{D}\mu = 2^4\,\mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathrm{d}^2\eta(1+\tfrac{3}{2}a\eta^2) \tag{3.28}$$

$$\mathscr{D}\mu^{\mathrm{R}} = 2^4\,\mathrm{d}^4x^{\mathrm{R}}\,\mathrm{d}^2\bar{\theta}^{\mathrm{R}}(1+\tfrac{3}{2}a\bar{\theta}^{\mathrm{R}^2}) \tag{3.29}$$

$$\mathscr{D}\mu^{\mathrm{L}} = 2^4\,\mathrm{d}^4x^{\mathrm{L}}\,\mathrm{d}^2\theta^{\mathrm{L}}. \tag{3.30}$$

After inserting the ϕ^{R} and ϕ^{L} into (3.27), and performing integration over anticommuting variables we obtain

$$\begin{aligned}\mathscr{L} = {} & \partial_\mu A^{\mathrm{R}}\partial^\mu A^{\mathrm{L}} + \tfrac{1}{2}\mathrm{i}(\psi^{A\mathrm{L}}\sigma^\mu{}_{A\dot{B}}\partial_\mu\bar{\psi}^{\dot{B}\mathrm{R}} + \mathrm{R}\leftrightarrow\mathrm{L}) + F^{\mathrm{R}}F^{\mathrm{L}} + aA^{\mathrm{R}}F^{\mathrm{L}} \\ & + M(A^{\mathrm{L}}F^{\mathrm{L}} - \tfrac{1}{2}\psi^{\mathrm{L}^2} + \mathrm{R}\leftrightarrow\mathrm{L}) + \tfrac{3}{2}aMA^{\mathrm{R}^2} \\ & + \sqrt{2}g(A^{\mathrm{L}^2}F^{\mathrm{L}} - A^{\mathrm{L}}\psi^{\mathrm{L}^2} + \mathrm{R}\leftrightarrow\mathrm{L}) + a\sqrt{2}gA^{\mathrm{R}^3}.\end{aligned} \tag{3.31}$$

The fields $F^{\mathrm{R}}, F^{\mathrm{L}}$ have no derivative terms in and may be therefore eliminated by virtue of their 'motion equations'

$$F^{\mathrm{R}} + MA^{\mathrm{L}} + \sqrt{2}gA^{\mathrm{L}^2} + aA^{\mathrm{R}} = 0 \tag{3.32}$$

$$F^{\mathrm{L}} + MA^{\mathrm{R}} + \sqrt{2}gA^{\mathrm{R}^2} = 0. \tag{3.33}$$

The final form of the Lagrangian that we obtain after the substitution

$$A^{\mathrm{L}} = 2^{-1/2}(A+\mathrm{i}B), \qquad A^{\mathrm{R}} = 2^{-1/2}(A-\mathrm{i}B) \tag{3.34}$$

$$\psi = \begin{pmatrix}\psi^{\mathrm{L}}_A \\ \bar{\psi}^{\dot{A}\mathrm{R}}\end{pmatrix} \tag{3.35}$$

where, however, A and B are still complex fields, is

$$\begin{aligned}\mathscr{L} = {} & \tfrac{1}{2}(\partial_\mu A)^2 + \tfrac{1}{2}(\partial_\mu B)^2 + \tfrac{1}{2}\mathrm{i}\bar{\psi}\gamma^\mu\partial_\mu\psi - \tfrac{1}{2}(M^2 - aM)A^2 - \tfrac{1}{2}(M^2 + aM)B^2 - \tfrac{1}{2}M\bar{\psi}\psi \\ & - gMA(A^2+B^2) - \tfrac{1}{2}g^2(A^2+B^2)^2 - g\bar{\psi}(A - B\gamma^5)\psi + \mathrm{i}MaAB.\end{aligned} \tag{3.36}$$

This Lagrangian differs from the Wess–Zumino Lagrangian of the standard supersymmetry in terms proportional to a and by the fact that the fields A, B are not real.

4. Conclusions

The contraction procedure proves to be a convenient and powerful tool for the construction of Casimir operators, superfield representations, and invariant measures for Konopelchenko's superalgebra, and may be applied without more serious difficulties.

Konopelchenko's superalgebra is related in different ways to other superalgebras used in physics (OSp(1, 4), supersymmetry GL algebra, superconformal algebra) but its own applications for the construction of physical models encounters serious problems. The source of the problem is the non-reality of the algebra. It prevents representing the odd generators by Hermitian operators and makes the K-symmetric Lagrangian non-real. Unfortunately we have not found any reasonable way to circumvent these difficulties.

Acknowledgments

I should like to thank J Niederle for bringing my attention to these themes and steady interest about the progress of the work. Useful discussions with M Bednář, E A Ivanov and L Trlifaj, as well as the helpful comments of V I Ogievetskij, are gratefully acknowledged.

Appendix. Spin-tensor algebra

Due to the fact that the odd generators Q_A, $R_{\dot{A}}$ are not connected by an involution there is no reason for considering them components of a Majorana bispinor but rather as two independent Weyl spinors. There is a standard procedure transforming tensors of the Lorentz group to Weyl spinors and spin-tensors by virtue of the matrices $\sigma^{\mu}{}_{A\dot{B}}$, $\sigma^{\mu\nu}{}_{AB}$, $\overline{\sigma^{\mu\nu}}{}_{\dot{C}\dot{D}}$ defined as

$$\sigma^{\mu}{}_{A\dot{B}} = \{\mathbb{1}, \sigma^{i}\} \tag{A1}$$

where σ^{i} are Pauli matrices and

$$\sigma^{\mu\nu}{}_{AB} = \tfrac{1}{2}(\sigma^{\mu}{}_{A\dot{C}}\sigma^{\nu}{}_{B}{}^{\dot{C}} - \sigma^{\nu}_{A\dot{C}}\sigma^{\mu}{}_{B}{}^{\dot{C}}) \tag{A2}$$

$$\overline{\sigma^{\mu\nu}}{}_{\dot{A}\dot{B}} = \tfrac{1}{2}(\sigma^{\nu D}{}_{\dot{A}}\sigma^{\nu}{}_{D\dot{B}} - \sigma^{\nu D}{}_{\dot{A}}\sigma^{\mu}{}_{D\dot{B}}). \tag{A3}$$

Indices $A, B, \ldots, \dot{A}, \dot{B}, \ldots$ run to 1, 2 and are raised and lowered by matrices

$$\mathscr{C}_{AB} = \mathscr{C}_{\dot{A}\dot{B}} = \mathscr{C}^{AB} = \mathscr{C}^{\dot{A}\dot{B}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A4}$$

We let

$$X^{A} = \mathscr{C}^{AB}X_{B}, \qquad X_{A} = X^{B}\mathscr{C}_{BA}. \tag{A5}$$

References

Fayet P and Ferrera S 1977 *Phys. Rep.* **32** 69
Golfand Yu A and Lichtman E P 1971 *JETP Lett.* **13** 452 (in Russian)
—— 1972 in *Problems of Theoretical Physics* (in Russian) (Moscow: Nauka) p 37
Haag R, Lopuszanski J and Sohnius M 1975 *Nucl. Phys.* B **88** 257
Hlavatý L and Niederle J 1980 *Lett. Math. Phys.* **4** 301
Ivanov E A and Sorin A S 1980 *J. Phys. A: Math. Gen.* **13** 1159
Jarvis P D and Green H S 1979 *J. Math. Phys.* **20** 2121
Konopelchenko B G 1975 *JETP Lett.* **21** 612 (in Russian)
Ogievetskij V I and Mezincescu L 1975 *Usp. Fiz. Nauk* **117** 637
Pais A and Rittenberg V 1975 *J. Math. Phys.* **16** 2062
Penrose R 1975 *Quantum Gravity, An Oxford Symposium* (Oxford: Clarendon)
Salam A and Strathdee J 1974 *Nucl. Phys.* B **76** 477
Schweber S S 1961 *An Introduction to Relativistic Quantum Field Theory* (Illinois, New York: Row Peterson)
Wess J and Zumino B 1974a *Nucl. Phys.* B **70** 39
—— 1974b *Phys. Lett.* **49B** 52
Zumino B 1977 *Nucl. Phys.* B **127** 189